Murat Kologlu, Gene S. Kopp, Steven J. Miller, and Yinghui Wang
On the Number of Summands in Zeckendorf Decompositions,
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Abstract

Zeckendorf proved that every positive integer has a unique representation as a sum of non-consecutive Fibonacci numbers. Once this has been shown, it's natural to ask how many summands are needed. Using a continued fraction approach, Lekkerkerker proved that the average number of such summands needed for integers in $[F_n, F_{n+1})$ is $n/(\alpha^2+1)+O(1)$, where F_n is the nth Fibonacci number and $\alpha = \frac{1+\sqrt{5}}{2}$ is the golden mean. Surprisingly, no one appears to have investigated the distribution of the number of summands; our main result is that this converges to a Gaussian as $n \to \infty$. Moreover, such a result holds not just for the Fibonacci numbers but many other problems, such as linear recurrence relations with non-negative integer coefficients (which is a generalization of base B expansions of numbers) and far-difference representations.

In general, the proofs involve adopting a combinatorial viewpoint and analyzing the resulting generating functions through partial fraction expansions and differentiating identities. The resulting generating functions through partial fraction expansions and differentiating identities. The resulting arguments become quite technical. The purpose of this paper is to concentrate on the special and most interesting case of the Fibonacci numbers, where the obstructions vanish and the proofs follow from some combinatorics and Stirling's formula; see [13] for proofs in the general case.