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Abstract

This paper generalizes a result of Gerdemann to show (with slight variations in some special cases) that, for any real number m and Horadam function $H_n(A, B, P, Q)$,

$$mH_n(A, B, P, Q) = \sum_{i=h}^{k} t_i H_{n+i}(A, B, P, Q),$$

for two consecutive values of n, if and only if,

$$m = \sum_{i=h}^{k} t_i a^i = \sum_{i=h}^{k} t_i b^i$$

where $a = \frac{P+\sqrt{P^2-4Q}}{2}$ and $b = \frac{P-\sqrt{P^2-4Q}}{2}$. (Horadam functions are defined by: $H_0(A, B, P, Q) = A$, $H_1(A, B, P, Q) = B$, $H_{n+1}(A, B, P, Q) = PH_n(A, B, P, Q) - QH_{n-1}(A, B, P, Q)$.) Further generalizations to the solutions of arbitrary linear recurrence relations are also considered.