John B. Cosgrave and Karl Dilcher

Abstract

Using Pell equations and known solutions that involve Lucas sequences, we find all solutions of the reciprocal pair of quadratic congruences $p^2 \equiv \pm 1 \pmod{q}$, $q^2 \equiv \pm 1 \pmod{p}$ for odd primes p, q. In particular, we show that there is exactly one solution $(p, q) = (3, 5)$ when the right-hand sides are -1 and 1. When the right-hand sides are both -1, there are four known solutions, all of them pairs of Fibonacci primes, and when the right-hand sides are both 1, there are no solutions. By partly different methods we completely characterize the solutions of $p^2 \equiv \pm N \pmod{q}$, $q^2 \equiv \pm N \pmod{p}$ for $N = 2$ and 4, and give partial results for $N = 3$ and 5. In the process we indicate how the general case can be treated.