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Sharper Upper Bounds for the Order of Appearance in the Fibonacci Sequence,
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Abstract

Let F_n be the nth Fibonacci number. The order of appearance $z(n)$ of a natural number n is defined as the smallest natural number k such that n divides F_k. In 1975, J. Sallé proved that $z(n) \leq 2n$, for all positive integers n. In this paper, we shall provide sharper upper bounds for $z(n)$ which are substantially smaller than $2n$ for some values of n. Moreover, we shall prove that

$$\liminf_{n\to\infty} \frac{z(n)}{n} = 0.$$