Abstract

For each positive integer n, we consider the following sequence of numbers
$$F(n), \quad F(nF(n)), \quad F(nF(nF(n))), \quad \ldots,$$
where $F(n)$ is the nth Fibonacci number defined in the usual way. Let $G_k(n)$ be the kth term of this sequence. We prove that $F(n)^k \parallel G_k(n)$ for all positive integers k and n with $n > 3$. For the first nontrivial case when $n = 3$, we prove that $F(3)^{2k-1} \parallel G_k(3)$ for all positive integers k. We also provide an alternative proof of the divisibility of $G_k(n)$ by $F(n)^k$ first proved by two authors of this work. Finally, we give explicit formulas of the quotients obtained from dividing $G_k(n)$ by $F(n)^k$ for the cases when $k = 2$ and $k = 3$.