Philippe Demontigny, Thao Do, Archit Kulkarni, Steven J. Miller, and Umang Varma
A Generalization of Fibonacci Far-Difference Representations and Gaussian Behavior,
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Abstract

A natural generalization of base B expansions is Zeckendorf's Theorem, which states that every integer can be uniquely written as a sum of non-consecutive Fibonacci numbers $\{F_n\}$, with $F_{n+1} = F_n + F_{n-1}$ and $F_1 = 1, F_2 = 2$. If instead we allow the coefficients of the Fibonacci numbers in the decomposition to be zero or ± 1, the resulting expression is known as the far-difference representation. Alpert proved that a far-difference representation exists and is unique under certain restraints that generalize non-consecutiveness, specifically that two adjacent summands of the same sign must be at least 4 indices apart and those of opposite signs must be at least 3 indices apart.

In this paper we prove that a far-difference representation can be created using sets of Skipponacci numbers, which are generated by recurrence relations of the form $S^{(k)}_{n+1} = S^{(k)}_n + S^{(k)}_{n-k}$ for $k \geq 0$. Every integer can be written uniquely as a sum of the $\pm S^{(k)}_n$'s such that every two terms of the same sign differ in index by at least $2k+2$, and every two terms of opposite signs differ in index by at least $k+2$. Let $I_n = (R_k(n-1), R_k(n)]$ with $R_k(\ell) = \sum_{0<\ell-b(2k+2)\leq \ell} S^{(k)}_{\ell-b(2k+2)}$. We prove that the number of positive and negative terms in given Skipponacci decompositions for $m \in I_n$ converges to a Gaussian as $n \to \infty$, with a computable correlation coefficient. We next explore the distribution of gaps between summands, and show that for any k the probability of finding a gap of length $j \geq 2k+2$ decays geometrically, with decay ratio equal to the largest root of the given k-Skipponacci recurrence. We conclude by finding sequences that have an (s,d) far-difference representation (see Definition 1.11) for any positive integers s, d.