

Patrick Flanagan, Marc S. Renault, and Josh Updike  
*Symmetries of Fibonacci Points, Mod  $m$ ,*  
Fibonacci Quart. **53** (2015), no. 1, 34–41.

**Abstract**

Given a modulus  $m$ , we examine the set of all points  $(F_i, F_{i+1}) \in \mathbb{Z}_m^2$  where  $F$  is the usual Fibonacci sequence. We graph the set in the fundamental domain  $[0, m-1] \times [0, m-1]$ , and observe that as  $m$  varies, sometimes the graph appears as a random scattering of points, but often it shows striking symmetry. We prove that in exactly three cases ( $m = 2, 3$ , or  $6$ ) the graph shows symmetry by reflection across the line  $y = x$ . We prove that symmetry by rotation occurs exactly when the terms  $0, -1$  appear half-way through a period of  $F \pmod{m}$ . We prove that symmetry by translation can occur in essentially one way, and we provide conditions equivalent to the graph having symmetry by translation.