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On primes in Lucas sequences,
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Abstract

Consider the Lucas sequence $u(a,b) = \{u_n(a,b)\}$ and the companion Lucas sequence $v(a,b) = \{v_n(a,b)\}$ which both satisfy the second order recursion relation

$$w_{n+2} = aw_{n+1} - bw_n$$

with initial terms $u_0 = 0$, $u_1 = 1$, and $v_0 = 2$, $v_1 = a$, respectively. We give both necessary and sufficient tests and also necessary tests for the primality of $|u_n|$ and $|v_n|$. For those tests which are only necessary, we show that these tests are not sufficient by means of a simple criterion using the Legendre symbol. These results are specialized to the Fibonacci numbers $\{F_n\}$ and to the Lucas numbers $\{L_n\}$. In particular, we generalize a result of Drobot giving criteria for F_p not to be prime, where p is a prime, to the Lucas numbers $\{L_n\}$.