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Abstract

Let p be an odd prime and let $u(a,1)$ and $u(a',1)$ be two Lucas sequences whose discriminants have the same nonzero quadratic character modulo p and whose periods modulo p are equal. We prove that there is then an integer c such that for all $d \in \mathbb{Z}_p$, the frequency with which d appears in a full period of $u(a,1) \pmod{p}$ is the same frequency as cd appears in $u(a',1) \pmod{p}$. Here $u(a,1)$ satisfies the recursion relation $u_{n+2} = au_{n+1} + u_n$ with initial terms $u_0 = 0$ and $u_1 = 1$. Similar results are obtained for the companion Lucas sequences $v(a,1)$ and $v(a',1)$. We also explicitly determine the exact distribution of residues of $u(a,1) \pmod{p}$ when $u(a,1)$ has a maximal period modulo p.