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Abstract

Zeckendorf's Theorem states that any positive integer can be written uniquely as a sum of non-adjacent Fibonacci numbers; this result has been generalized to many recurrence relations, especially those arising from linear recurrences with leading term positive. We investigate legal decompositions arising from two new sequences: the (s, b)-Generacci sequence and the Fibonacci Quilt sequence. Both satisfy recurrence relations with leading term zero, and thus previous results and techniques do not apply. These sequences exhibit drastically different behavior. We show that the (s, b)-Generacci sequence leads to unique legal decompositions, whereas not only do we have non-unique legal decompositions with the Fibonacci Quilt sequence, we also have that in this case the average number of legal decompositions grows exponentially. Another interesting difference is that while in the (s, b)-Generacci case the greedy algorithm always leads to a legal decomposition, in the Fibonacci Quilt setting the greedy algorithm leads to a legal decomposition (approximately) 93% of the time. In the (s, b)-Generacci case, we again have Gaussian behavior in the number of summands as well as for the Fibonacci Quilt sequence when we restrict to decompositions resulting from a modified greedy algorithm.