Abstract

The binary Fibonacci sequence is the sequence of 0's and 1's obtained by starting from 0 and iterating in parallel the substitution rules $0 \to 01$, $1 \to 0$ infinitely many times: the first construction steps successively yield the binary strings 0, 01, 010, 01001, ... (whose lengths are $1, 2, 3, 5, ...$). This sequence is in some sense one of the "simplest" non- periodic sequences. It can also be obtained by playing billiard on a square. In this survey we describe some "extremal" properties of the binary Fibonacci sequence and of similar sequences (the *Sturmian* sequences): in particular we recall unexpected inequalities involving these sequences and their shifted sequences.