Ray Li and Steven J. Miller
A Collection of Central Limit Type Results in Generalized Zeckendorf Decompositions,
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Abstract

Zeckendorf's Theorem states that if the Fibonacci numbers are indexed as $F_1 = 1$, $F_2 = 2$, $F_3 = 3$, $F_4 = 5, \dots,$ then every positive integer can be written un iquely as the sum of non-adjacent Fibonacci numbers. This result can be generalized to certain classes of linear recurrence relations $\{G_n\}$ with appropriate notions of decompositions. For many decompositions, the distribution of the number of summands in the decomposition of an $M \in [G_n, G_{n+1})$ is known to converge to a Gaussian as $n \to \infty$. This work discusses a more general approach to proving this kind of asymptotic Gaussian behavior that also bypasses technical obstructions in previous approaches. The approach is motivated by the binomials $a_{n,k} = \binom{n}{k}$. The binomials satisfy the recursion $a_{n,k} = a_{n-1,k} + a_{n-1,k-1}$ and are well known to have the property that the random variables $\{X_n\}_{n=1}^{\infty}$ given by $\Pr[X_n = k] = a_{n,k} / \sum_{i=0}^{\infty} a_{n,i}$ converge to a Gaussian as $n \to \infty$. This new approach proves that appropriate two-dimensional recurrences exhibit similar asymptotic Gaussian behavior. From this, we can reprove that the number of summands in decompositions given by many linear recurrence relations is asymptotically Gaussian and additionally prove that for any non-negative integer g, the number of gaps of size g in the decomposition of an $M \in [G_n, G_{n+1})$ also converges to a Gaussian as $n \to \infty$.