R. S. Melham
Further Closed Forms for Finite Sums of Weighted Products of the Sine and Cosine Functions,
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Abstract

In this paper, we present closed forms for six families of finite sums of weighted products of the sine/cosine functions. In each finite sum that we define, the summand contains a product of trigonometric functions, and the length of this product can be made as large as we please.

A special case of one of our main results is the sum

$$\sum_{i=1}^{n} \left(\frac{1}{2\cos 2}\right)^{i} \cos i \cos(i-3) = \cos 1 - \frac{\cos n \cos(n+1)}{2^{n} \cos^{n} 2}.$$

Here the weight term in the summand is $\left(\frac{1}{2\cos 2}\right)^{i}$.