Andreas N. Philippou and Spiros D. Dafnis
A Simple Proof of an Identity Generalizing Fibonacci-Lucas Identities,
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Abstract

Let $F_n^{(k)} = 0$ for $-k+1 \leq n \leq 0$, $F_1^{(k)} = 1$, and $F_n^{(k)} = \sum_{j=1}^{k} F_{n-j}^{(k)}$ for $n \geq 2$. Also let $L_0^{(k)} = k$, $L_1^{(k)} = 1$, $L_n^{(k)} = n + \sum_{j=1}^{n-1} L_{n-j}^{(k)}$ for $2 \leq n \leq k$, and $L_n^{(k)} = \sum_{j=1}^{k} L_{n-j}^{(k)}$ for $n \geq k+1$. The identity $\sum_{i=0}^{n} m^i \Big(\big(L_i^{(k)} + (m-2) F_{i+1}^{(k)} - \sum_{j=3}^{k} (j-2) F_{i-j+1}^{(k)} \big) \Big) = m^{n+1} F_{n+1}^{(k)} + k - 2$ $(m \geq 2, k \geq 2)$, derived recently by means of colored tiling [4], is presently proved using only the definitions of $F_n^{(k)}$ and $L_n^{(k)}$, and the identity $L_n^{(k)} = \sum_{j=1}^{k} j F_{n-j+1}^{(k)}$ $(n \geq 1)$.