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Generalizations of Hermite's Identity and Applications,
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Abstract

Hermite's identity states that

$$\sum_{0\leq k\leq n-1}\left\lfloor x+\frac{k}{n}\right\rfloor=\lfloor nx\rfloor \text{ for all } x\in\mathbb{R} \text{ and } n\in\mathbb{N}.$$

In this article, we give a generalization of this identity and show some applications. For example, we consider the above sum when k ranges over the integers from a to b, where $a<b$ are integers. Then, we apply it to give another proof of a recent result of Tverberg. We also obtain a formula for the corresponding sum when k ranges over a complete residue system modulo n.