Eric Chen, Robin Chen, Lucy Guo, Cindy Jiang, Steven J. Miller, Joshua M. Siktar, and Peter Yu
Gaussian Behavior in Zeckendorf Decompositions from Lattices,
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Abstract

Zeckendorf's Theorem states that any positive integer can be written uniquely as a sum of nonadjacent Fibonacci numbers. We consider higher-dimensional lattice analogues in a chosen dimension $d \geq 1$, where a legal decomposition of a number n is a collection of lattice points such that each point is included at most once. Once a point is chosen, all future points must have strictly smaller coordinates, and the pairwise sum of the values of the points chosen equals n. We prove that the distribution of the number of summands in these lattice decompositions converges to a Gaussian distribution in d dimensions. As an immediate corollary, we obtain a new proof for the asymptotic number of certain lattice paths.