Ray Li and Steven J. Miller
Central Limit Theorems for Gaps of Generalized Zeckendorf Decompositions,
Fibonacci Quart. **57** (2019), no. 3, 213–230.

Abstract

Zeckendorf proved that every integer can be written uniquely as a sum of nonadjacent Fibonacci numbers $\{1, 2, 3, 5, \dots\}$. This has been extended to many other recurrence relations $\{G_n\}$ (with their own notion of a legal decomposition). It has also been proved that the distribution of the number of summands of an $M \in [G_n, G_{n+1})$ converges to a Gaussian as $n \to \infty$. We prove that for any nonnegative integer g, the average number of gaps of size g in many generalized Zeckendorf decompositions is $C_\mu n + d_\mu + o(1)$ for constants $C_\mu > 0$ and d_μ depending on g and the recurrence, the variance of the number of gaps of size g is similarly $C_\sigma n + d_\sigma + o(1)$ for constants $C_\sigma > 0$ and d_σ, and the number of gaps of size g of an $M \in [G_n, G_{n+1})$ converges to a Gaussian as $n \to \infty$. We show this by proving a general result on when an associated two-dimensional recurrence converges to a Gaussian, and additionally re-derive other results in the literature.