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Some High Degree Generalized Fibonacci Identities,
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Abstract

The Gelin-Cesáro identity states that for integers $n \geq 2$,

$$F_{n-2}F_{n-1}F_{n+1}F_{n+2} - F_n^4 = -1,$$

where $\{F_n\}$ is the Fibonacci sequence. Horadam generalized the Fibonacci sequence by defining the sequence $\{W_n\}$ where $W_0 = a$, $W_1 = b$, and $W_n = pW_{n-1} - qW_{n-2}$ for $n \geq 2$ and a, b, p and q are integers and $q \neq 0$. Using this sequence, Melham and Shannon generalized the Gelin-Cesáro identity by proving that for integers $n \geq 2$,

$$W_{n-2}W_{n-1}W_{n+1}W_{n+2} - W_n^4 = cq^{n-2}(p^2 + q)W_n^2 + c^2q^{2n-3}p^2,$$

where $c = pab - qa^2 - b^2$. We will discover and prove some similar high degree generalized Fibonacci identities.