Steven J. Miller and Alexandra Newlon
The Fibonacci Quilt Game,
Fibonacci Quart. **58** (2020), no. 2, 157–168.

Abstract

Zeckendorf [13] proved that every positive integer can be expressed as the sum of nonconsecutive Fibonacci numbers. This theorem inspired a beautiful game, the Zeckendorf Game [2]. Two players begin with n 1's and take turns applying rules inspired by the Fibonacci recurrence, $F_{n+1} = F_n + F_{n-1}$, until a decomposition without consecutive terms is reached; whoever makes the last move wins. We look at a game resulting from a generalization of the Fibonacci numbers, the Fibonacci Quilt sequence [3]. This sequence arises from the two-dimensional geometric property of tiling the plane through the Fibonacci spiral. Beginning with 1 in the center, we place integers in the squares of the spiral such that each square contains the smallest positive integer that does not have a decomposition as the sum of previous terms that do not share a wall. This sequence eventually follows two recurrence relations, allowing us to construct a variation on the Zeckendorf Game, the Fibonacci Quilt Game. Whereas some properties of the Fibonacci sequence are inherited by this sequence, the nature of its recurrence leads to others, such as Zeckendorf's theorem, no longer holding. Thus, it is of interest to investigate the generalization of the game in this setting to see which behaviors persist. We prove, similar to the original game, that this game also always terminates in a legal decomposition. We give a lower bound on game lengths, showing that, depending on strategies, the length of the game can vary and either player could win. Finally, we give a conjecture on the length of a random game.