Hùng Việt Chu, Steven J. Miller, and Zimu Xiang
Higher Order Fibonacci Sequences from Generalized Schreier Sets,
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Abstract

A Schreier set S is a subset of the natural numbers with $\min S \geq |S|$. It has been known that the sequence $(a_{1,n})$, where

$$a_{1,n} \;=\; |\{S \subseteq \mathbb{N} \,:\, \max S = n \text{ and } \min S \geq |S|\}|$$

is the Fibonacci sequence. Generalizing this result, we prove that for all $p \in \mathbb{N}$, the sequence $(a_{p,n})$, where

$$a_{p,n} \;=\; |\{S \subseteq \mathbb{N} \,:\, \max S = n \text{ and } \min S \geq p|S|\}|$$

has a linear recurrence relation of higher order. We investigate further by requiring that $\min_2 S \geq q|S|$, where $\min_2 S$ is the second smallest element of S. We prove a linear recurrence relation for the sequence $(a_{p,q,n})$, where

$$a_{p,q,n} \;=\; |\{S \subseteq \mathbb{N} \,:\, \max S = n,\ \min S \geq p|S|,\ \text{and } \min_2 S \geq q|S|\}|,$$

and discuss a curious relationship between $(a_{q,n})$ and $(a_{p,q,n})$.