Evan Fang, Jonathan Jenkins, Zack Lee, Daniel Li, Ethan Lu, Steven J. Miller, Dilhan Salgado, and Joshua M. Siktar

Central Limit Theorems for Compound Paths on the Two-Dimensional Lattice,

Fibonacci Quart. **58** (2020), no. 3, 208–225.

Abstract

Zeckendorf proved that every integer can be written uniquely as a sum of nonconsecutive Fibonacci numbers $\{F_n\}$, with later researchers showing that the distribution of the number of summands needed for such decompositions of integers in $[F_n, F_{n+1})$ converges to a Gaussian as $n \to \infty$. Decomposition problems have been studied extensively for a variety of different sequences and notions of legal decompositions; for the Fibonacci numbers, a legal decomposition is one for which each summand is used at most once and no two consecutive summands may be chosen. Chen, et al. [11] generalized earlier work to d-dimensional lattices of positive integers; there, a legal decomposition was defined as a path such that every point chosen had each component strictly less than the same component of the previous chosen point in the path. They were able to prove Gaussianity results despite the lack of uniqueness of the decompositions; however, one would expect their results to hold in the more general case where some components are identical. The strictly decreasing assumption was needed in that work to obtain simple, closed form combinatorial expressions, which could then be well approximated and lead to the limiting behavior. In this work, we remove that assumption through inclusion-exclusion arguments. These lead to more involved combinatorial sums; using generating functions and recurrence relations, we obtain tractable forms in two dimensions and prove Gaussianity again. A more involved analysis should work in higher dimensions.