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Representation of $\frac{1}{2}(F_n - 1)(F_{n+1} - 1)$ *and* $\frac{1}{2}(F_n - 1)(F_{n+2} - 1)$,
Fibonacci Quart. **58** (2020), no. 4, 334–339.

Abstract

Let $a, b \in \mathbb{N}$ be relatively prime. We consider $(a-1)(b-1)/2$, which arises in the study of the pqth cyclotomic polynomial, where p, q are distinct primes. We prove two possible representations of $(a-1)(b-1)/2$ as nonnegative integral linear combinations of a and b. Surprisingly, for each pair (a, b), only one of the two representations exists and the representation is also unique. We then investigate the representations of $(F_n-1)(F_{n+1}-1)/2$ and $(F_n-1)(F_{n+2}-1)/2$, where F_i is the ith Fibonacci number, and observe several nice patterns.