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*A New Combinatorial Interpretation of the Fibonacci Numbers Cubed*,
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**Abstract**

We consider the tiling of an $n$-board (an $n \times 1$ rectangular board) with third-squares ($\frac{1}{3} \times 1$ tiles with the shorter sides always aligned horizontally) and $(\frac{1}{3}, \frac{2}{3})$-fence tiles. A $(w, g)$-fence tile is composed of two $w \times 1$ subtiles separated by a $g \times 1$ gap. We show that the number of ways to tile an $n$-board using these types of tiles equals $F_{n+1}^3$ where $F_n$ is the $n$th Fibonacci number. We use these tilings to devise straightforward combinatorial proofs of identities relating the Fibonacci numbers cubed to one another, to other combinations of Fibonacci numbers, and to the Pell numbers. Some of these identities appear to be new. We also show that for $p = 2, 3, \ldots$, the number of ways to tile an $n$-board using either $1/p \times 1$ tiles and $(1/p, 1 - 1/p)$-fences or $(1/2p, 1/2 - 1/2p)$- and $(1/2p, 1 - 1/2p)$-fences is $F_{n+1}^p$.