

Burghard Herrmann
The Continued Fraction Pendulum,
Fibonacci Quart. **58** (2020), no. 5, 144–151.

Abstract

For an irrational number α let $\langle i\alpha \rangle$ denote the fractional part of $i\alpha$ where i is any integer. The three distance theorem states that any t points $\langle i\alpha \rangle$, $1 \leq i \leq t$, partition the unit interval into gaps of at most three distinct lengths. We know that the process of splitting gaps for increasing t swings like an escalating pendulum in the unit interval and we show that the margins are determined by the denominators of the convergents of the continued fraction representation of α .

Moreover, for a positive real number ξ the points $(\langle i\alpha \rangle, i/\xi)$ provide a strip of a lattice. The main result states that the smallest distance between lattice points is determined by a denominator of a principal convergent. Regarding this and the second smallest distance, lattices are classified into a landscape of phyllotactic patterns.