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Girard-Waring Type Formula for a Generalized Fibonacci Sequence,
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Abstract

Let $f(x) = x^k + a_1 x^{k-1} + \cdots + a_k$ be a monic polynomial of degree $k \geq 2$ with distinct roots $\{x_i | i = 1, \ldots, k\}$. Let $f'(x)$ be the derivative of $f(x)$, $P_n = x_1^n/f'(x_1) + x_2^n/f'(x_2) + \cdots + x_k^n/f'(x_k)$ and $Q_n = x_1^n + x_2^n + \cdots + x_k^n$; P_n is a generalized Fibonacci sequence and Q_n is a generalized Lucas sequence. We have a Girard-Waring type formula for P_n:

$$P_n = \sum_{j_1,\ldots,j_k} (-a_1)^{j_1} (-a_2)^{j_2} \cdots (-a_k)^{j_k} \cdot \frac{(j_1 + j_2 + \cdots + j_k)!}{j_1! j_2! \cdots j_k!}$$

where the indices $j_1, j_2, \ldots, j_k$ satisfy $j_1 + 2j_2 + \cdots + kj_k = n - k + 1$. We have formulas for the generating function for P_n, and Q_n:

$$G_P(x) = (1/x)/f(1/x), \quad G_Q(x) = (1/x)f'(1/x)/f(1/x).$$