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Second-Order Linear Recurrences Having Arbitrarily Large Defect Modulo p ,
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Abstract

Let $(w) = w(a, b)$ denote the second-order linear recurrence satisfying $w_{n+2} = aw_{n+1} + bw_n$, where w_0, w_1, and a are integers, $b = \pm 1$, and $D = a^2 + 4b$ is the discriminant. We distinguish the Lucas sequences $u(a, b)$ and $v(a, b)$ with initial terms $u_0 = 0$, $u_1 = 1$, and $v_0 = 2$, $v_1 = a$, respectively. Let p be a prime. Given the recurrence $w(a, b)$, let $\delta_w(p)$, called the *defect* of $w(a, b)$ modulo p, denote the number of residues not appearing in (w) modulo p. It is known that for the recurrence $w(a, \pm 1)$, $\delta_w(p) \geq 1$ if $p > 7$ and $p \nmid D$. Given the fixed recurrence $w(a, 1)$, where $w(a, 1) = u(a, 1)$ or $v(a, 1)$, we will show that $\lim_{p\to\infty} \delta_w(p) = \infty$. Further, given the arbitrary recurrence $w(a, -1)$, we will demonstrate that $\lim_{p\to\infty} \delta_w(p) = \infty$ and $\lim_{p\to\infty} \delta_w(p)/p \geq \frac{1}{2}$. We will also prove that for the arbitrary recurrence $w(a, \pm 1)$, we have that $\limsup_{p\to\infty} \delta_w(p)/p = 1$.