Amelia Gilson, Hadley Killen, Tamás Lengyel, Steven J. Miller, Nadia Razek, Joshua M. Siktar, and Liza Sulkin
Zeckendorf's Theorem Using Indices in an Arithmetic Progression, Fibonacci Quart. **59** (2021), no. 4, 327–337.

Abstract

Zeckendorf's Theorem states that any positive integer can be uniquely decomposed into a sum of distinct, nonadjacent Fibonacci numbers. There are many generalizations, including results on existence of decompositions using only even indexed Fibonacci numbers. We extend these further and prove that similar results hold when only using indices in a given arithmetic progression. As part of our proofs, we generate a range of new recurrences for the Fibonacci numbers that are of interest in their own right.