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A Method for Uniformly Proving a Family of Identities,
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Abstract

This paper presents a proof method and a result. The proof method presented is particularly suitable for uniformly proving families of identities satisfied by a family of recursive sequences. To illustrate the method, we study the family of recursive sequences $F_n^{(k)} = \sum_{i=1}^{k} F_{n-i}^{(k)}$, $n \geq 0$, $k \geq 2$, with n a parameter varying over integers, and k a parameter indexing members of the family. The main theorem states $F_n^{(k)} = \sum_{j=1}^{k} P_{k,j} F_{n-jk}^{(k)}$, with P a recursive triangle satisfying the triangle recursion $P_{i,j} = 2P_{i-1,j} - P_{i-1,j-1}$, with appropriate initial conditions. The proof of the theorem exploits the fact that characteristic polynomials of identities are divisible by the characteristic polynomial of the recursion generating the underlying sequence.