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Three Cousins of Recamán's Sequence,
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Abstract

Although 10^{230} terms of Recamán's sequence have been computed, it remains a mystery. Here three distant cousins of that sequence are described, one of which is also mysterious. (i) $\{A(n), n \geq 3\}$ is defined as follows. Start with n, and add $n+1$, $n+2$, $n+3, \ldots$, stopping after adding $n+k$ if the sum $n+(n+1)+\ldots+(n+k)$ is divisible by $n+k+1$. Then $A(n)=k$. We determine $A(n)$ and show that $A(n) \leq n^2-2n-1$. (ii) $\{B(n), n \geq 1\}$ is a multiplicative analog of $\{A(n)\}$. Start with n, and successively multiply by $n+1$, $n+2, \ldots$, stopping after multiplying by $n+k$ if the product $n(n+1)\cdots(n+k)$ is divisible by $n+k+1$. Then $B(n)=k$. We conjecture that $\log^2 B(n) = (\frac{1}{2}+o(1)) \log n \log\log n$. (iii) The third sequence, $\{C(n), n \geq 1\}$, is the most interesting, because it is the most mysterious. Concatenate the decimal digits of $n, n+1, n+2, \ldots$ until the concatenation $n \| n+1 \| \ldots \| n+k$ is divisible by $n+k+1$. Then $C(n)=k$. If no such k exists, we set $C(n)=-1$. We have found k for all $n \leq 1000$ except for two cases. Some of the numbers involved are quite large. For example, $C(92) = 218128159460$, and the concatenation $92 \| 93 \| \ldots \| (92+C(92))$ is a number with about $2 \cdot 10^{12}$ digits. We have only a probabilistic argument that such a k exists for all n.