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Abstract

Consider the generating function (gf) for the integer sequence $(F_{mi}F_{ni} : i \in \mathbb{N}_0)$, where m and n are positive integer parameters. We may compute this gf in terms of Fibonacci/Lucas numbers using an implementation of an algorithm by Zeilberger. However, for the case where the integers m and n have the same parity, we have experimentally discovered that there is a simpler way of expressing this gf compared with the corresponding expression obtained via Zeilberger's procedure. We prove this equivalence via Binet's formula, and then apply our simplified gf evaluation to generalize a classic Fibonacci sum identity given by Freitag, and some order-2 Fibonacci-type sums in the recent work of Melham. Our evaluations of finite sums over $F_{mi}F_{ni}$ are dramatically simpler compared with the corresponding output obtained via Zeilberger's `Cfinite` Maple package.