Benjamin Baily, Justine Dell, Irfan Durmić, Henry L. Fleischmann,
Faye Jackson, Isaac Mijares, Steven J. Miller, Ethan Pesikoff, Luke
Reifenberg, Alicia Smith Reina, and Yingzi Yang

Abstract

Every positive integer may be written uniquely as a base-φ decomposition–that is a legal sum of powers of φ, the golden mean. Guided by earlier work on a two-player game which produces the Zeckendorf Decomposition of an integer, we define a related game played on an infinite tuple of non-negative integers which decomposes a positive integer into its base-φ expansion. We call this game the Bergman Game. We prove that the longest possible Bergman game on an initial state S with n summands terminates in $\Theta(n^2)$ time, and we also prove that the shortest possible Bergman game on an initial state terminates in $\Theta(n)$ time. We also show a linear bound on the maximum length of the tuple used throughout the game.