Hao Cui, Xiaoyu Cui, Sophia C. Davis, Irfan Durmić, Qingcheng Hu, Lisa Liu, Steven J. Miller, Fengping Ren, Alicia Smith Reina, and Eliel Sosis
Sums of Reciprocals of Recurrence Relations,
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Abstract There is a growing literature on sums of reciprocals of polynomial functions of recurrence relations with constant coefficients and fixed depth, such as Fibonacci and Tribonacci numbers, products of such numbers, and balancing numbers (numbers n such that the sum of the integers less than n equals the sum of the r integers immediately after, for some r which is called the balancer of n; If n is included in the summation, we have the cobalancing numbers, and r is called the cobalancer of n). We generalize previous work to reciprocal sums of depth two recurrence sequences with arbitrary coefficients and the Tribonacci numbers, and show our method provides an alternative proof of some existing results.

We define (a, b) balancing and cobalancing numbers, where a and b are constants that multiply the left-hand side and right-hand side respectively, and derive recurrence relations describing these sequences. We show that for balancing numbers, the coefficients $(3, 1)$ is unique such that every integer is a $(3, 1)$ balancing number, and proved there does not exist an analogous set of coefficients for cobalancing numbers. We also found patterns for certain coefficients that have no balancing or cobalancing numbers.