aBa Mbirika and Jürgen Spilker
GCD of sums of k consecutive squares of generalized Fibonacci numbers,
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Abstract

In 2021, Guyer and Mbirika gave two equivalent formulas that computed the greatest common divisor (GCD) of all sums of k consecutive terms in the generalized Fibonacci sequence $(G_n)_{n\geq 0}$ given by the recurrence $G_n = G_{n-1} + G_{n-2}$ for all $n \geq 2$ with integral initial conditions G_0 and G_1. In this current paper, we extend their results to the GCD of all sums of k consecutive squares of these numbers. Denoting these GCD values by the symbol $\mathcal{G}^2_{G_0,G_1}(k)$, we prove $\mathcal{G}^2_{G_0,G_1}(k) = \gcd\left(G_kG_{k+1} - G_0G_1,\ G^2_{k+1} - G^2_1,\ G^2_{k+2} - G^2_2\right)$. Moreover, we provide very tantalizing closed forms in the specific settings of the Fibonacci, Lucas, and generalized Fibonacci numbers. We close with a number of open questions for further research.