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Recurrence relations for S-legal index difference sequences,
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Abstract

Zeckendorf's Theorem implies that the Fibonacci number F_n is the smallest positive integer that cannot be written as a sum of nonconsecutive previous Fibonacci numbers. Catral et al. studied a variation of the Fibonacci sequence, the Fibonacci Quilt sequence: the plane is tiled using the Fibonacci spiral, and integers are assigned to the squares of the spiral such that each square contains the smallest positive integer that cannot be expressed as the sum of nonadjacent previous terms. This adjacency is essentially captured in the differences of the indices of each square: the ith and jth squares are adjacent if and only if $|i-j| \in \{1,3,4\}$ or $\{i,j\} = \{1,3\}$.

We consider a generalization of this construction: given a set of positive integers S, the S-legal index difference (S-LID) sequence $(a_n)_{n=1}^{\infty}$ is defined by letting a_n be the smallest positive integer that cannot be written as $\sum_{\ell \in L} a_\ell$ for some set $L \subset [n-1]$ with $|i-j| \notin S$ for all $i, j \in L$. We discuss our results governing the growth of S-LID sequences, as well as results proving that many families of sets S yield S-LID sequences that follow simple recurrence relations.