ADDITIONAL FACTORS OF THE FIBONACCI AND LUCAS SERIES

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In his volume entitled, "Recurring Sequences," D. Jarden (1) has listed known factors of the first 385 Fibonacci and Lucas numbers. The present article has for purpose to explore these numbers for additional factors in the range $p < 3000$.

Initially recourse was had to the results of D. D. Wall (2). His table lists all primes less than 2,000 which have a period other than the maximum. A comparison of these results with Jarden's factorizations indicated that the following additional factors are now known.

FACTOR	NUMBER	FACTOR	NUMBER
1279	L(213)	1823	L(304)
1523	L(254)	1871	L(187)
1553	F(259)	1877	F(313)
1579	L(263)	1913	F(319)
1699	L(283)	1973	F(329)
1733	F(289)	1999	L(333)

The altered factorizations are given below, the newly introduced factors being starred while the remaining residual factors are underlined.

$$L(213) = (2^2)\ (1279^*)\ (688846502588399)\ (\underline{92750098539536589172558519})$$

$$L(254) = (3)\ (1523)^*\ (\underline{26487408254541486132499495083633739300801481688547})$$

$$F(259) = (13)\ (73)(149)\ (1553*)(2221)\ (\underline{1230669188181354229694664202889707409030657})$$

$$L(263) = (1579)*\ (\underline{58259567431970886012123727669192696712910749985451 01})$$

L(283) = (1699*) (81904697726943126494463230440143683491348547590190211271)

F(289) = (577)(1597)(1733*) (699300337863809553116509146296699696041517688627857)

L(304) = (607)(1823*)(2207) (13946490749426062743697525919851871606820418027874934441)

L(187) = (199)(1871*)(3571)(9056742344085065262650973 90431)

F(313) = (1877*)(6168536281287720504015660343294357770749152912304487547909 0829)

F(319) = (89)(1913*)(514229)(2373070801850309840641893568419180819509608713746211 3977)

F(329) = (13)(1973*)(2971215073) (33530815263744997367320800108983382828522283904656580 77)

L(333) = (2^2) (19) (1999*)(4441)(146521)(1121101)(54018521) (65416866960304807819786581576757029610 6159)

The next step was to explore the periods of all primes in the range 2000 < p < 3000. In carrying out this work the following points were kept in mind:

(1) For primes of the form 10x ± 1, the period k(p) of the Fibonacci series is a factor of p-1; for primes of the form 10x ± 3, k (p) is a factor of 2p + 2.

(2) For primes of the form 10x ± 1, the period is even; for primes of the form 10x ± 3, the period has the same power of 2 as is found in 2p + 2.

The first zero of the Fibonacci series for a prime p is indicated by Z(F, p) , while the first zero for the Lucas series is denoted Z(L, p).

All cases are covered by the following:

(1) If k(p) is of the form 2(2y + 1), then Z(f, p) = k(p)/4 and Z(L, p) = k(p)/2.

(2) If k(p) is of the form $2^2(2y + 1)$, then Z(F, p) = k(p)/4 and p is not a factor of the Lucas series.

(3) If k(p) is of the form $2^m(2y + 1)$, $m \geq 3$, then Z(F, p) is k(p)/2 and Z(L, p) is k(p)/4.

It is to be noted that in the first case Z(L, p) is odd and that Z(F, p) is likewise odd in the second case. In the third case Z(L, p) is even. Knowledge of the first zeros in the Fibonacci and Lucas series leads to a direct conclusion regarding the period of the Fibonacci series.

In the table that follows, the period of the Fibonacci series as well as the first zeros in the Fibonacci and Lucas series are given for all primes in the range $2000 < p < 3000$. All Fibonacci numbers with index a multiple of Z(F, p) have the given prime as a divisor; all Lucas numbers with index an odd multiple of Z(L, p) have the given prime as a divisor.

TABLE OF PERIODS AND ZEROS OF THE FIBONACCI AND LUCAS SERIES IN THE RANGE 2000 to 3000

p	k(p)	Z(F, p)	Z(L, p)
2003	4008	2004	1002
2011	2010	2010	1005
2017	4036	1009	----
2027	1352	676	338
2029	1014	1014	507

TABLE OF PERIODS AND ZEROS

p	k(p)	Z(F,p)	Z(L,p)
2039	2038	2038	1019
2053	4108	1027	----
2063	4128	2064	1032
2069	1034	1034	517
2081	130	130	65
2083	4168	2084	1042
2087	4176	2088	1044
2089	1044	261	----
2099	2098	2098	1049
2111	2110	2110	1055
2113	4228	1057	----
2129	2128	1064	532
2131	2130	2130	1065
2137	4276	1069	----
2141	2140	535	----
2143	4288	2144	1072
2153	4308	1077	----
2161	80	40	20
2179	198	198	99
2203	4408	2204	1102
2207	64	32	16
2213	4428	1107	----
2221	148	37	----
2237	1492	373	----
2239	746	746	373
2243	4488	2244	1122
2251	750	750	375
2267	1512	756	378
2269	324	81	----
2273	4548	1137	----

TABLE OF PERIODS AND ZEROS

p	k(p)	Z(F,p)	Z(L,p)
2281	760	380	190
2287	4576	2288	1144
2293	4588	1147	----
2297	4596	1149	----
2309	2308	577	----
2311	2310	2310	1155
2333	1556	389	----
2339	2338	2338	1169
2341	2340	585	----
2347	4696	2348	1174
2351	2350	2350	1175
2357	4716	1179	----
2371	790	790	395
2377	4756	1189	----
2381	2380	595	----
2383	4768	2384	1192
2389	398	398	199
2393	4788	1197	----
2399	2398	2398	1199
2411	2410	2410	1205
2417	124	31	----
2423	4848	2424	1212
2437	4876	1219	----
2441	1220	305	----
2447	1632	816	408
2459	2458	2458	1229
2467	4936	2468	1234
2473	4948	1237	----
2477	4956	1239	----
2503	5008	2504	1252

TABLE OF PERIODS AND ZEROS

p	k(p)	Z(F, p)	Z(L, p)
2521	120	60	30
2531	2530	2530	1265
2539	2538	2538	1269
2543	5088	2544	1272
2549	2548	637	----
2551	2550	2550	1275
2557	5116	1279	----
2579	2578	2578	1289
2591	518	518	259
2593	5188	1297	----
2609	2608	1304	652
2617	5236	1309	----
2621	1310	1310	655
2633	5268	1317	----
2647	5296	2648	1324
2657	5316	1329	----
2659	886	886	443
2663	1776	888	444
2671	2670	2670	1335
2677	5356	1339	----
2683	5368	2684	1342
2687	1792	896	448
2689	896	448	224
2693	5388	1347	----
2699	2698	2698	1349
2707	5416	2708	1354
2711	2710	2710	1355
2713	5428	1357	----
2719	2718	2718	1359
2729	682	682	341

TABLE OF PERIODS AND ZEROS

p	k(p)	Z(F,p)	Z(L,p)
2731	390	390	195
2741	2740	685	----
2749	916	229	----
2753	1836	459	----
2767	5536	2768	1384
2777	1852	463	----
2789	164	41	----
2791	2790	2790	1395
2797	5596	1399	----
2801	1400	700	350
2803	5608	2804	1402
2819	2818	2818	1409
2833	5668	1417	----
2837	5676	1419	----
2843	5688	2844	1422
2851	2850	2850	1425
2857	5716	1429	----
2861	1430	1430	715
2879	2878	2878	1439
2887	5776	2888	1444
2897	5796	1449	----
2903	5808	2904	1452
2909	2908	727	----
2917	5936	1459	----
2927	5856	2928	1464
2939	2938	2938	1469
2953	5908	1477	----
2957	5916	1479	----
2963	5928	2964	1482
2969	424	212	106
2971	2970	2970	1485
2999	2998	2998	1499

The revised factorizations of Fibonacci numbers resulting from the information in the preceding table are listed below.

F(229) = (457)(2749*)(256799205151071273644115114294714688654853)

F(261) = (2) (17)(173)(2089*)(514229)(3821263937)(65082172574960442149015615136409)

F(305) = (5) (2441*)(4513)(555003497) (806206763478084210952216884085652444453430427 61)

F(373) = (2237*)(179159081379972024769599382295527508296287028193832492595657294050015005389)

The revised factorizations of Lucas series numbers resulting from the present investigation are given herewith.

L(190) = (3) (41) (2281*)(29134601)(6240396376418455747284925 21)

L(199) = (2389*) (1623021451704572904627621714280893312 41)

L(224) = (1087)(2689*)(4481)(496633631041375772840631751560627532 9)

L(259) = (29) (2591*)(54018521)(330666900546898116460968438563218940272271)

L(338) = (3) (2027*)(90481)(78938707151259468242664283515154949332581380084893707250688723)

L(341) = (199)(2729*) (3010349) (112541283906252545479268192813140395290253805955295249179369)

L(350) = (3) (41) (281) (401) (2801*)(57061)(12317523121)
(51256536896717129910976518385167664503518615201)

L(373) = (2239*)(40025403581523031569114917729520068273516094100108818109883490482374673 8439)

L(378) = $(2)\ (3^4)$(83)(107)(281)(1427)(2267*)(11128427)
(335411542061 5683)(61077153262397604948064467593047434562 9523)

CONCLUSION

The present work has continued the table of Wall in systematically determining the periods of primes beyond 2000 and less than 3000. In addition, information has been provided regarding the first zeros of the Fibonacci and Lucas series in this same region.

As a result of Wall's data and its extension in this paper, additional factorizations have been found for Fibonacci and Lucas numbers as given in Jarden's tables. In particular L (263), F(313), F(373), L(199), and L(373) which were previously unfactored have now been shown to be composite.

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