ELEMENTARY PROBLEMS AND SOLUTIONS

EDITED BY S. L. BASIN, SAN JOSE STATE COLLEGE

Send all communications regarding Elementary Problems and Solutions to S. L. Basin, 946 Rose Ave., Redwood City, California. We welcome any problems believed to be new in the area of recurrent sequences as well as new approaches to existing problems. The proposer must submit his problem with solution in legible form, preferably typed in double spacing, with the name(s) and address of the proposer clearly indicated. Solutions should be submitted within two months of the appearance of the problems.

B-9 *Proposed by R. L. Graham, Bell Telephone Laboratories, Murray Hill, New Jersey*

Prove
$$\sum_{n=2}^{\infty} \frac{1}{F_{n-1} F_{n+1}} = 1$$

and
$$\sum_{n=2}^{\infty} \frac{F_n}{F_{n-1} F_{n+1}} = 2 ,$$

where F_n is the nth Fibonacci number.

B-10 *Proposed by Stephen Fisk, San Francisco, California*

Prove the "de Moivre-type" identity,

$$\left(\frac{L_n + \sqrt{5}\, F_n}{2}\right)^p = \frac{L_{np} + \sqrt{5}\, F_{np}}{2} ,$$

where L_n denotes the nth Lucas number and F_n denotes the nth Fibonacci number.

B-11 *Proposed by S. L. Basin, Sylvania Electronic Defense Laboratory, Mt. View, California*

Show that the hypergeometric function

$$G(x, n) = \sum_{k=0}^{n-1} \frac{2^k (n + k)!\ (x - 1)^k}{(n - k - 1)!\ (2k + 1)!}$$

generates the sequence $G\left(\frac{3}{2}, n\right) = F_{2n}$, $\qquad n = 1, 2, 3, \cdots .$

B-12 *Proposed by Paul F. Byrd, San Jose State College, San Jose, Calif.*

Show that

$$L_{n+1} = \begin{vmatrix} 3 & i & 0 & 0 & \cdots & 0 & 0 \\ i & 1 & i & 0 & \cdots & 0 & 0 \\ 0 & i & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & i & 1 & \cdots & 0 & 0 \\ . & . & . & . & \cdots & . & . \\ 0 & 0 & 0 & 0 & \cdots & 1 & i \\ 0 & 0 & 0 & 0 & \cdots & i & 1 \end{vmatrix}_n \qquad n \geq 1$$

where L_n is the nth Lucas number given by $L_1 = 1$, $L_2 = 3$, $L_{n+2} = L_{n+1} + L_n$, and $i = \sqrt{-1}$.

B-13 *Proposed by S. L. Basin, Sylvania Electronic Defense Laboratory, Mt. View, Calif.*

Determinants of order n which are of the form,

$$K_n(b, c, a) = \begin{vmatrix} c & a & 0 & 0 & 0 & \\ b & c & a & 0 & 0 & \cdots \\ 0 & b & c & a & 0 & \cdots \\ 0 & 0 & b & c & a & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix}_n$$

are known as <u>CONTINUANTS</u>.

Prove that,

$$K_n(b, c, a) = \frac{(c + \sqrt{c^2 - 4\,ab})^{n+1} - (c - \sqrt{c^2 - 4\,ab})^{n+1}}{2^{n+1}\sqrt{c^2 - 4\,ab}}$$

and show, for special values of a, b, and c, that $K_n(b, c, a) = F_{n+1}$.

B-14 *Proposed by Maxey Brooke, Sweeny, Texas, and C. R. Wall, Ft. Worth, Texas*

Show that

$$\sum_{n=1}^{\infty} \frac{F_n}{10^n} = \frac{10}{89} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1} F_n}{10^n} = \frac{10}{109} .$$

B-15 *Proposed by R. B. Wallace, Beverly Hills, Calif. and Stephen Geller, University of Alaska, College, Alaska.*

If p_k is the smallest positive integer such that

$$F_{n+p_k} \equiv F_n \bmod (10^k)$$

for all positive n, then p_k is called the period of the Fibonacci sequence relative to 10^k. Show that p_k exists for each k, and find a specific formula for p_k as a function of k.

B-16 *Proposed by Marjorie Bicknell, San Jose State College, and Terry Brennan, Lockheed Missiles & Space Co., Sunnyvale, Calif.*

Show that if

$$R = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix},$$

then

$$R^n = \begin{pmatrix} F_{n-1}^2 & F_{n-1}F_n & F_n^2 \\ 2F_{n-1}F_n & F_{n+1}^2 - F_{n-1}F_n & 2F_nF_{n+1} \\ F_n^2 & F_nF_{n+1} & F_{n+1}^2 \end{pmatrix}.$$

NOTE: On occasion there will be problems listed at the ends of the articles in the advanced and elementary sections of the magazine. These problems are to be considered as logical extensions of the corresponding problem sections and solutions for these problems will be discussed in these sections as they are received.

See, for example, "Expansion of Analytic Functions In Polynomials Associated with Fibonacci Numbers," by Paul F. Byrd, San Jose State College, in the first issue of the Quarterly, and "Linear Recurrence Relations—Part I," by James Jeske, San Jose State College, in this issue.

Solutions for problems in ISSUE ONE will appear in ISSUE THREE.