EXPLORING THE FIBONACCI REPRESENTATION OF INTEGERS

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Every integer may be represented as the sum of Fibonacci numbers or as a single such number. What is being considered in this investigation is the smallest number of different Fibonacci numbers required in the representation of an integer. For example, 125 is the sum of $89 + 34 + 2$. This seems to be the smallest number of Fibonacci numbers required to represent 125.

The following question is being proposed: Is it possible to set up an upper limit to this minimum number of Fibonacci numbers required to represent any integer? Possibly, there are many approaches to a solution, but one particular line of development will be indicated here.

We need first of all some notation. A well known symbol is the square bracket [] which means "the greatest integer in." Thus

$$[6.3] = 6; \qquad [5] = 5; \qquad [17/3] = 5 \quad .$$

Along with this we are going to introduce a similar notation to mean "the greatest Fibonacci number in." Thus

$$[63]^* = 55; \qquad [189/4]^* = 34; \qquad [13]^* = 13 \quad .$$

One way to solve the proposed question may be indicated by the following partially stated theorem:

Theorem. The maximum number of different Fibonacci numbers required to represent an integer N for which $[N]^* = F_n$ is given by [] . The answer in the bracket is some function of n. Explorers who find this result are encouraged to report their solution. In addition, there is a line of proofs that could be formulated to show that the theorem holds in general.

The above investigation will be reported in the April, 1964, issue of the Fibonacci Quarterly.