RECIPROCALS OF GENERALIZED FIBONACCI NUMBERS

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One of the oldest procedures for the numerical solution of $f(x) = 0$ is the classical regula falsi method. This "rule of false position" is given by the iteration

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})} ,$$

where x_1 and x_2 are the initial estimates. (It may be noted that the regula falsi method is simply inverse linear interpolation.)

For the innocuous equation $x^2 = 0$, this iteration reduces to

$$x_{n+1} = \frac{x_{n-1} x_n}{x_{n-1} + x_n} .$$

If we define the generalized Fibonacci numbers by

$$F_1 = a, \; F_2 = b, \; F_3 = a + b, \; F_4 = a + 2b, \cdots$$
$$F_{n+2} = F_{n+1} + F_n, \cdots$$

it immediately follows that with starting values $x_1 = 1/a$, $x_2 = 1/b$, this application of regula falsi yields the reciprocals of the generalized Fibonacci numbers since

$$\frac{\frac{1}{F_{i+1}} \cdot \frac{1}{F_{i+2}}}{\frac{1}{F_{i+1}} + \frac{1}{F_{i+2}}} = \frac{1}{F_{i+1} + F_{i+2}} = \frac{1}{F_{i+3}} .$$