 [Continued from page 42.]

(4.15) $$A_r(x) = \sum_{n=1}^{F_{r-1}} x^{a^2(n)+n-2} .$$

Note that by (4.15) and (4.3), we have

$$\lim_{r\to\infty} A_r(x) = x^{-1}\psi_1(x)$$

in agreement with (4.12) and (4.14).

Exactly as in [1] it can be shown that the function $\psi_1(x)$ has the unit circle for a natural boundary. In view of (4.12) the same is true of each of the functions $\psi_k(x)$.

It would be of interest to know whether there is any simple relation connecting $\psi_1(x)$ with

$$\phi(x) = \sum_{n=1}^{\infty} x^{a(n)} .$$

In particular, do there exist polynomials $P(x)$, $Q(x)$, $R(x)$ such that

(4.16) $$P(x)\phi(x) + Q(x)\psi_1(x) = R(x) \text{ ?}$$

5. FURTHER RESULTS

In [3] the following are given:

$\nu(kL_n) = kF_{n-1}$, for n sufficiently large;

$\nu(5kF_n) = kL_{n-1}$, for n sufficiently large;

$\nu(L_{2n}^2) = F_{4n-1}$, $(n \ge 1)$; $\nu(L_{2n-1}^2) = F_{4n-3} - 1$, $(n \ge 1)$;

$\nu(F_{2n}) = F_nF_{n-1}$, $(n \ge 2)$; $\nu(L_nL_{n-1}) = F_{2n-2}$, $(n \ge 2)$;

$\nu(L_{2n+1}L_{2n-1}) = F_{4n-1} - 1$, $(n \ge 1)$; $\nu(L_{2n+2}L_{2n}) = F_{4n+1} + 1$, $(n \ge 1)$;

$\nu(5F_n) = L_{n-1}$, $(n \ge 2)$; $\nu(5F_n^2) = F_nL_{n-1}$, $(n \ge 3)$;

$\nu(5F_nF_{n+1}) = F_{2n}$, $(n \ge 1)$; $\nu(5F_{2n}F_{2n-2}) = F_{4n-3} - 1$, $(n \ge 1)$.

[Continued on page 112.]