In each case, the generalized Pascal's array can be generated by adding all the elements in the rectangle with k rows above and to the left of element A (not including elements in the same column as A) to get A. If the rectangle has k rows, then we get the array induced by the expansions $(1 + x + x^2 + \cdots + x^{k-1})^n$, $n = 0, 1, 2, \cdots$. In these rectangular arrays using k rows in formation, if sums are found of elements lying on diagonals formed by going up $(k + 1)$ and right one, the sequence formed obeys the recurrence

$$u_{n+k+1} = u_{n+k} + u_{n+k-1} + \cdots + u_n \; .$$

where $u_1 = u_2 = 1$, $u_n = 2^{n-2}$ for $2 \le n \le k + 1$, generalized Fibonacci sequences, while the rising diagonals yield sums which are generalized Pell sequences obeying the recurrence

$$p_{n+k} = 2p_{n+k-1} + (p_{n+k-2} + p_{n+k-3} + \cdots + p_n)$$

and with the first three members of the sequence the ordinary Pell numbers 1, 2, 5, and the first k members of the sequence the same as the first k members of the sequence found from the rectangular array using $(k - 1)$ rows in its formation.

The convolution triangle for such generalized Fibonacci sequences can be generated by adding all the elements in the rectangle with k rows, including the column above an element A and extending to the extreme left of the array.

In any of these generalized Pascal's arrays or convolution arrays of generalized Fibonacci sequences written in rectangular form, the determinant of any square array found in the upper left-hand corner is always equal to one. The proofs and extensions will appear in later papers [2], [3].

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