

( $m = 0, 1, 2, \dots, n$ ), where  $F_{-2} = F_{-1} = 0$  and  $F_j$  for each  $j \geq 0$  is the  $j^{\text{th}}$  Fibonacci number (2).

The results in Theorem 2 were suggested to the authors by considering a number of special cases on an IBM 360/65 computer.

## REFERENCES

1. R. L. Duncan, "Note on the Euclidean Algorithm," The Fibonacci Quarterly, Vol. 4 (1966), pp. 367-368.
2. O. Perron, Die Lehre von den Kettenbrüchen, Vol. 1, Teubner, Stuttgart, 1954.
3. J. V. Uspensky and M. A. Heaslet, Elementary Number Theory, McGraw-Hill, 1939.



## LETTERS TO THE EDITOR

Dear Editor:

In the paper (\*) by W. A. Al-Salam and A. Verma, "Fibonacci Numbers and Eulerian Polynomials," Fibonacci Quarterly, February 1971, pp. 18-22, an error occurs in (9), which is readily corrected. I will generalize their (4) by defining a general polynomial operator  $M$  by

$$(I) \quad Mf(x) = Af(x + c_1) + Bf(x + c_2), \quad c_1 \neq c_2,$$

where  $f(x)$  is a polynomial and  $A, B, c_1,$  and  $c_2$  are given numbers. With  $D = d/dx$ , we note that  $M = Ae^{c_1 D} + Be^{c_2 D}$  so that

$$Mf(x) = A \sum_{n=0}^{\infty} \frac{c_1^n}{n!} D^n f(x) + B \sum_{n=0}^{\infty} \frac{c_2^n}{n!} D^n f(x),$$

or

$$(II) \quad Af(x + c_1) + Bf(x + c_2) = \sum_{n=0}^{\infty} \frac{W_n}{n!} D^n f(x),$$

where  $W_n = Ac_1^n + Bc_2^n$  is the solution of  $W_{n+2} = PW_{n+1} - QW_n$  and  $c_1 \neq c_2$  are the roots of  $x^2 = Px - Q$ . In (\*), Eq. (4) is a special case of (I) with  $A = \mu$  and  $B = 1 - \mu$ . There are two cases of (II) to consider:

Case 1.  $A + B \neq 0$ . If  $A = B$ , we obtain from (II)

$$(III) \quad f(x + c_1) + f(x + c_2) = \sum_{n=0}^{\infty} \frac{V_n}{n!} D^n f(x),$$

where  $V_0 = 2, V_1 = P,$  and  $V_{n+2} = PV_{n+1} - QV_n$ . If  $c_1$  and  $c_2$  are roots of  $x^2 = x + 1,$   
[Continued on page 71.]