

where  $s_n$  is the  $n^{\text{th}}$  triangular-square number.

Likewise, we can compute a formula for the  $n^{\text{th}}$  triangular-pentagonal number. The result is

$$s_n = \frac{(2 - \sqrt{3})(97 + 56\sqrt{3})^n + (2 + \sqrt{3})(97 - 56\sqrt{3})^n - 4}{48} .$$

This agrees with a result recently published by W. Sierpiński [4].

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#### REFERENCES

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[Continued from page 71.]

$$M^{-1} = \sum_{k=0}^{\infty} \frac{m_k^*}{k!} D^k$$

is given by

$$(VII) \quad \sum_{k=0}^{\infty} \frac{m_k^*}{k!} t^k = 1/(Ae^{c_1 t} + Be^{c_2 t}) .$$

We now note that for Case 2, where  $A + B = 0$ , Eq. (VII) does not exist for  $t = 0$ , and hence there is no inverse operator  $M^{-1}$ . Thus, a sufficient condition for  $M^{-1}$  (see (I)) to exist is that  $A + B \neq 0$ , i. e., Case 1. For  $A + B \neq 0$ , one readily finds that

$$(VIII) \quad (A + B)m_k^* = (c_2 - c_1)^k H_k \left( \frac{c_1}{c_1 - c_2} \middle| -A/B \right) ,$$

where  $H_k(x|\lambda)$  is the Eulerian polynomial cited in (\*).

Many more identities can be quoted. Indeed, for  $m, n = 0, 1, \dots$ , one has

[Continued on page 112.]