PHI: ANOTHER HIDING PLACE

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From an area A of any outline, regular or irregular, there is cut an area B, having the same outline as that of A under the following conditions: (i) The peripheries of A and B have one point O in common; (ii) B is oriented so that O and the centroids C_a and C_b of A and B are colinear. It follows that C, the centroid of the remnant (A - B) also lies in the straight line OC_aC_b produced.

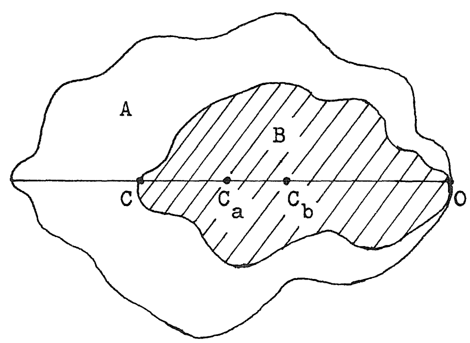

Fig. 1

Let the ratio of the linear dimensions of A and B be a:b, their respective areas being λa^2, λb^2; $OC_a/OC_b = a/b$.

Taking moments about O,

$$\lambda b^2 \cdot OC_b + \lambda(a^2 - b^2) \cdot OC = \lambda a^2 \cdot OC_a \ ,$$

whence, multiplying by $1/\lambda b^2 \cdot OC_b$,

$$1 + \left(\frac{a^2}{b^2} - 1\right) \cdot \frac{OC}{OC_b} = \frac{a^3}{b^3} \ .$$

Since $(a/b) - 1 \neq 0$,

$$\left(\frac{a}{b} + 1\right) \cdot \frac{OC}{OC_b} = \frac{a^2}{b^2} + \frac{a}{b} + 1 \ .$$

Phi, the Golden Section, is now uncovered by writing $OC/OC_b = 2$, giving

$$\frac{a^2}{b^2} - \frac{a}{b} - 1 = 0 \ ,$$

whence $a/b = \phi$ or $a/b = 1/\phi$.

The result is, of course, applicable to regular plane figures. In the case of the circle the centroid C of the remnant lune falls on the endpoint of the diameter of B through O.

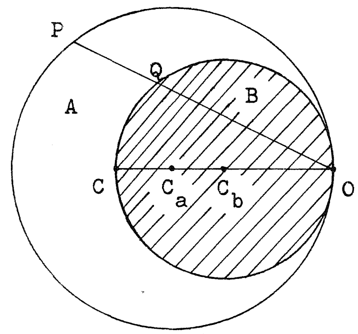

Fig. 2

Any chord of circle A through O is cut by the circumference of B in the Golden Section: $PO/QO = \phi = (1 + \sqrt{5})/2$.

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