FIBONACCIAN PATHOLOGICAL CURVES

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There are many curves which possess peculiar properties not possessed by ordinary curves. These are the so-called "pathological curves" of mathematics. In the present note a few curves which are not normal and healthy and which possess idiosyncrasies have been generated and analyzed. It may be pointed out that these curves cannot be analyzed with the help of ordinary calculus.

We generate Fibonaccian pathological curves as follows. Start with a square with side of length H_n, where H_n is the generalized Fibonacci number obtained by the recurrence relation

$$H_n = H_{n-1} + H_{n-2}, \qquad n > 2,$$

where H_1 and H_2 are any positive integers. Divide each side of the square into three parts, two of length H_{n-2} and one part of length H_{n-3}, as shown in Fig. 1.

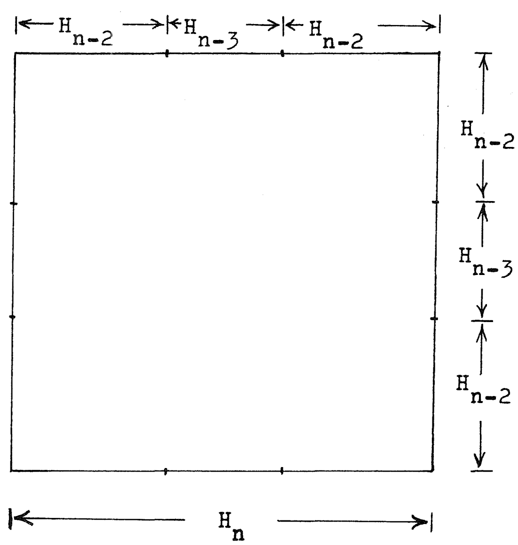

Fig. 1

On each section with length H_{n-3} erect a square outwards. Erase the basic side of this new figure and call this curve S_1. In S_1, shown in Fig. 2, we will have the sides of length H_{n-2} and H_{n-3}.

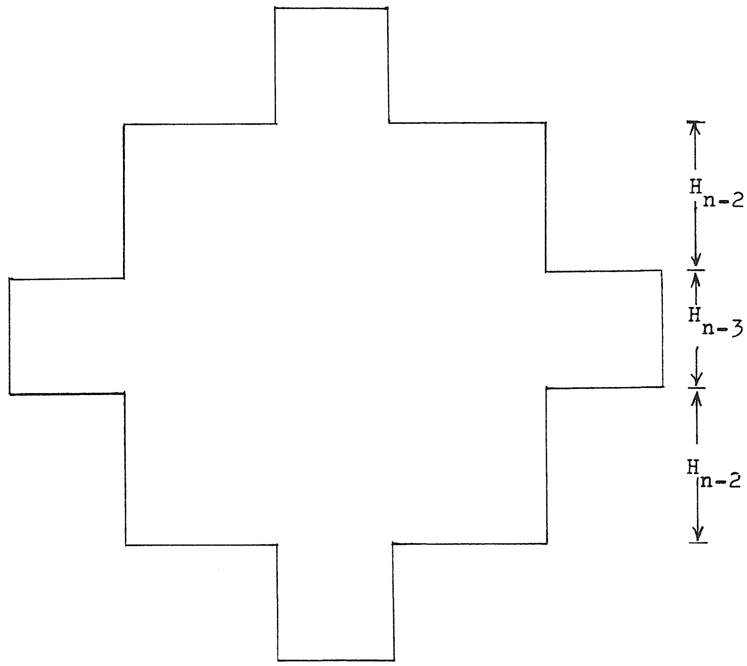

Fig. 2

Again, each side is divided into three parts. The sides of S_1 will be divided as follows:

$$H_{n-2} = H_{n-4} + H_{n-5} + H_{n-4} ,$$
$$H_{n-3} = H_{n-5} + H_{n-6} + H_{n-5} .$$

Again we will form a square on the middle part, forming curve S_2 as shown in Fig. 3.

We continue dividing each side into lengths equal to two lower Fibonacci numbers and constructing squares on the middle parts until finally we get sides of length H_2 and H_1, and have formed the curve S. We called the curve S the Fibonaccian Pathological Curve.

In such a construction, it is of interest to find the total length L_r and the rate of increase of area ΔA_r of the curve S_r at the r^{th} successive subdivision. It can be seen that, for $n > 3r$,

$$L_r = 4\left(2^r H_{n-2r} + \binom{r}{1} 2^{r-1} \cdot 3H_{n-2r-1} + \binom{r}{2} 2^{r-2} \cdot 3^2 H_{n-2r-2} + \cdots + 3^r H_{n-3r}\right)$$

$$\Delta A_r = 4\left(2^r H^2_{n-2r-3} + \binom{r}{1} 2^{r-1} \cdot 3H^2_{n-2r-4} + \binom{r}{2} 2^{r-2} \cdot 3^2 H^2_{n-2r-5} + \cdots + 3^r H^2_{n-3r-3}\right) .$$

It is of interest to note that H_1 and H_2 can be chosen as arbitrarily small positive numbers, and the curve after allowing all successive subdivisions will be a continuous curve which is not differentiable anywhere. It may be noted that an inwards curve can also be

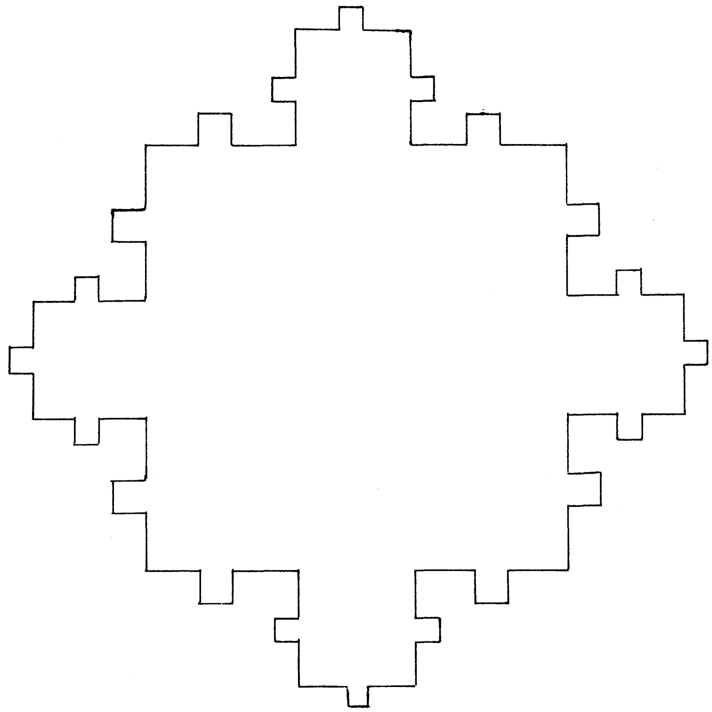

Fig. 3

generated on similar lines, but, due to lack of symmetry, the expression for obtaining the total area after r successive subdivisions is difficult to obtain.