Relying on the known result that the period of divisibility by m_1m_2 $(m_1, m_2$ co-prime) is given by $D(m_1m_2) = LCM(z_1, z_2)$ (see Wall [6]), we get the results:

$LCM(3,5) = 15$, and so F_{15} is the first Fibonacci number to be divisible by 10. $lcm(6,25) = 150$, and so F_{150} is divisible by 100. $LCM(12,625) = 7{,}500$ and so F_{7500} is divisible by 10^4.

This has been an exercise in finding the z numbers. By an extension of the argument we can produce the corresponding k numbers—the period of recurrence of the Fibonacci numbers (mod m^2).

REFERENCES

1. S.N. Collings, "Fibonacci Numbers," *Mathematics Teaching,* No. 52 (1970), p. 23.
2. D.E. Daykin and L.A.G. Dresel, "Factorization of Fibonacci Numbers," *The Fibonacci Quarterly,* Vol. 8, No. 1 (February 1970), pp. 23-30.
3. V.E. Hoggatt, Jr., *Fibonacci and Lucas Numbers,* Houghton-Mifflin, Boston, 1969, p. 59.
4. A.G. Marshall, "Fibonacci, Modulo n," *Mathematics Teaching,* No. 46 (1969), p. 29.
5. J. Vinson, "The Relation of the Period Modulo to the Rank of Apparition of m in the Fibonacci Sequence," *The Fibonacci Quarterly,* Vol. 1, No. 2 (April 1963), pp. 37–45.
6. D.D. Wall, "Fibonacci Series Modulo m," *American Math. Monthly,* Vol. 67 (1960), pp. 525–532.
7. O. Wyler, "On Second-Order Recurrences," *American Math. Monthly,* Vol. 72 (1965), pp. 500–506.

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$$F_k(x) = \sum_{j=0}^{[k/2]} (-1)^j e^j \frac{k}{k-j} \binom{k-j}{j} g_{k-2j}((-1)^j x) . \tag{5}$$

Write

$$\begin{cases} h_k(x) = (1 - a_k x + (-1)^k x^2) g_k(x) \\ c_k \quad = [(r-sb)a]^k + [(sa-r)b]^k . \end{cases} \tag{6}$$

Following Riordan [6], with $a_0 = 2$ and $h_0(x) = 1 - x$, we eventually derive

$$\begin{cases} c_1 + s\sqrt{5}\,x = h_1(x) \\ c_2 - x(2e + 5s^2) = h_2(x) - 2e\left\{h_0(-x) - (a_0 + a_2)xg_0(-x)\right\} \\ c_3 + s\sqrt{5}\,x(3e + 5s^2) = h_3(x) - 3e\left\{h_1(-x) - (a_1 + a_3)xg_1(-x)\right\} \\ c_4 - x(2e^2 + 20s^2e + 25s^4) = h_4(x) - 4e\left\{h_2(-x) - (a_2 + a_4)xg_2(-x)\right\} \\ \qquad + 2e^2\left\{h_0(x) - (a_4 - a_0)xg_0(x)\right\} \\ c_5 - e_1 = h_5(x) - 5e\left\{h_3(-x) - (a_3 + a_5)xg_3(-x)\right\} + 5e^2\left\{h_1(x) - (a_5 - a_1)xg_1(x)\right\} \end{cases} \tag{7}$$

where

$$e_1 = 2r^5 - 5r^4s + 30r^2s^2 - 40r^2s^3 + 35rs^4 - 10s^5 .$$

Substituting values of $a_k = a^k + b^k$, we have

$$\begin{cases} h_1(x) = \sqrt{5}\,(r + sx) \\ h_2(x) = 5(r^2 - s^2x) - 10exg_0(-x) \\ h_3(x) = 5\sqrt{5}\,(r^3 + s^3x) - 15exg_1(-x) \\ h_4(x) = 25(r^4 - s^4x) - 40exg_2(-x) + 50e^2xg_0(x) \\ h_5(x) = 25\sqrt{5}\,(r^5 + s^5x) - 75exg_3(-x) + 125e^2xg_1(x) . \end{cases} \tag{8}$$

These functions lead back to (2).

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