(70) $$G_{n+k}(x,y) = yH_{n-1}(x,y)V_k(x,y) + H_n(x,y)V_{k+1}(x,y).$$

Using (55) with (69) and (67), it can be shown that

(71) $$H_{n-1}(x,y)V_k(x,y) - H_n(x,y)V_{k-1}(x,y) = (-1)^k y^{k-1} G_{n-k}(x,y).$$

Letting k be odd or even in (68) through (71), we have

(72) $$H_{n+k}(x,y) + y^k H_{n-k}(x,y) = H_n(x,y)V_k(x,y), \quad k \text{ even};$$

(73) $$H_{n+k}(x,y) + y^k H_{n-k}(x,y) = G_n(x,y)U_k(x,y), \quad k \text{ odd};$$

(74) $$H_{n+k}(x,y) - y^k H_{n-k}(x,y) = H_n(x,y)V_k(x,y), \quad k \text{ odd};$$

(75) $$H_{n+k}(x,y) - y^k H_{n-k}(x,y) = G_n(x,y)U_k(x,y), \quad k \text{ even};$$

(76) $$G_{n+k}(x,y) + y^k G_{n-k}(x,y) = G_n(x,y)V_k(x,y), \quad k \text{ even};$$

(77) $$G_{n+k}(x,y) + y^k G_{n-k}(x,y) = (x^2+4y)H_n(x,y)U_k(x,y), \quad k \text{ odd};$$

(78) $$G_{n+k}(x,y) - y^k G_{n-k}(x,y) = G_n(x,y)V_k(x,y), \quad k \text{ odd};$$

(79) $$G_{n+k}(x,y) - y^k G_{n-k}(x,y) = (x^2+4y)H_n(x,y)U_k(x,y), \quad k \text{ even}.$$

Observe that if we replace H by U and G by V then Eqs. (72) through (79) yield Eqs. (56) through (63).

If we let $y = 1$ in (64) then Eqs. (72) through (79) are those of (30) through (33) and (36) through (39) where we replace $V_n(x,y)$ by L_n, $H_n(x,y)$ by H_n, $G_n(x,y)$ by G_n, and $U_n(x,y)$ by F_n. The same substitutions in (40) through (51) will give us the summation-product relations relative to the sequences $\{H_n(x,y)\}$ and $\{G_n(x,y)\}$ if $y = 1$.

In conclusion, we observe several other results which are a direct consequence of the formulas of this paper [2; p. 19].

If we replace n by $k+1$ in (5) through (8) we have F_k, L_k, F_{k+1}, and L_{k+1} are relatively prime to F_{2k+1} for $k \geq 1$. If we let $n = k+2$ in (5) through (8), we have F_k, L_k, F_{k+2}, and L_{k+2} are all relatively prime to F_{2k+2} for $k \geq 1$. Letting $n = k+1$ in (9) through (12), we see that F_k, L_k, F_{k+1}, and L_{k+1} are all relatively prime to L_{2k+1}.

If we let $n = k+1$ in (56) through (59) with $y = 1$ we see that the Fibonacci polynomials $U_{2k+1}(x,1) \pm 1$ are factorable for $k \geq 2$. If $n = k$ with $y = 1$ in (56) through (59) then $U_{2k}(x,1)$ is factorable for $k \geq 2$.

REFERENCES

1. V.E. Hoggatt, Jr., *Fibonacci and Lucas Numbers,* Houghton Mifflin Co., 1969.
2. V.E. Hoggatt, Jr., and G.E. Bergum, "Divisibility and Congruence Relations," *The Fibonacci Quarterly,* Vol. 12, No. 2 (April 1974), pp. 189–195.
3. A.F. Horadam, "A Generalized Fibonacci Sequence," *American Mathematical Monthly,* Vol. 66, 1959, pp. 445–459.
4. I. Niven and H. Zuckerman, *Introduction to the Theory of Numbers,* John Wiley and Sons, Inc., 1960.

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[Continued from Page 110.]

(3B) *If* k *is an integer for which Fermat's Last Theorem is true, then there is no pythagorean triangle with the hypotenuse and one of the legs equal to* kth *powers of natural numbers.*

Proofs of 1B and 2B are provided in the complete text, but 3B remains an open question.

The authors have attempted to compile a complete bibliography related to pythagorean triangles. Included in the bibliography are 111 references to journal articles, 66 references to problems (with solutions) in *Amer. Math Monthly,* 17 references to notes in *Math. Gaz.,* and 12 references to notes in *Math. Mag.* Since it is impossible to compile such a bibliography without some omissions, the authors would appreciate receiving any references not already included in the bibliography.

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