PRODUCTS AND POWERS

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The generalized Fibonacci sequence is defined by

(1) $$w_n = pw_{n-1} + qw_{n-2}$$

with

$$w_0 = a \quad \text{and} \quad w_1 = b.$$

In Horadam's notation [1], w_n is written $w_n(a,b;p,-q)$.

In this note we see what happens when we replace the sum and products in (1) by a product and powers; i.e.,

(2) $$z_n = z_{n-1}^p \cdot z_{n-2}^q$$

with

$$z_0 = a \quad \text{and} \quad z_1 = b.$$

(We can write z_n as $z_n(a,b;p,q)$.)

The sequence becomes $a, b, ab, ab^2, a^2b^3, a^3b^5, a^5b^8, \cdots$ in the case where $p = q = 1$ so that

$$z_n(a,b;1,1) = a^{F_{n-1}} \cdot b^{F_n}$$

The general case gives the sequence

$$a,\ b,\ a^pb^q,\ a^{pq},\ b^{p+q^2},\ a^{p^2+pq^2},\ b^{2pq+q^3},\ \cdots$$

with

$$z_n(a,b;p,q) = a^{w_n(1,0;p,-q)} \cdot b^{w_n(0,1;p,-q)}.$$

REFERENCE

1. A.F. Horadam, "Generating Functions for Powers of a Certain Generalized Sequence of Numbers," *Duke Math. Journal.*, Vol. 32, No. 3, pp. 437–446, Sept. 1965.

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