The proof of (*) for Fibonacci numbers is then completed by observing the relationships among these generating functions. For example,

$$\sum_{n=0}^{\infty}\left(F_{2n+m+2}^2 - F_m^2\right)x^n = \sum_{n=0}^{\infty} F_{2n+m+2}^2 x^n - F_m^2 \sum_{n=0}^{\infty} x^n$$

$$= \frac{F_{m+2}^2 + [(-1)^{m+2} - 3F_mF_{m+2}]x + F_m^2x^2}{(1-x)(1-7x+x^2)} - \frac{F_m^2}{1-x}$$

$$= \frac{(F_{m+2}^2 - F_m^2) + [(-1)^m - 3F_mF_{m+2} + 7F_m^2]x}{(1-x)(1-7x+x^2)}$$

$$= \frac{F_{2m+2} - F_{2m-2}x}{(1-x)(1-7x+x^2)}$$

$$= \sum_{n=0}^{\infty}\left(\sum_{k=0}^{2n+1} F_kF_{k+2m+1}\right)x^n ,$$

and hence,

$$\sum_{k=0}^{2n+1} F_kF_{k+2n+1} = F_{2n+m+2}^2 - F_m^2 .$$

The other three cases are similar.

REFERENCE

1. V. E. Hoggatt, Jr., and J. C. Anaya, "A Primer for the Fibonacci Numbers: Part XI," *The Fibonacci Quarterly*, Vol. 11, No. 1 (Feb., 1973), pp. 85–90.

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[Continued from P. 342.]

Proof. The corollary is known to be true for $(b/-1) = 1$. Then the following results can be calculated:

If

$$(a_1a_2/-1) = 1,$$

then

$$(a_1a_2/b) = 1,$$
$$(-a_1a_2/b) = (-1/b),$$
$$(a_1a_2/-b) = 1,$$
$$(-a_1a_2/-b) = -(-1/b);$$

If $(a_1a_2/-1) = -1$, then

$$(a_1a_2/b) = 1,$$
$$(-a_1a_2/b) = (-1/b),$$
$$(a_1a_2/-b) = -1,$$
$$(-a_1a_2/-b) = (-1/b).$$

[Continued on P. 349.]