$$= \sum_{m,s=0}^{\infty} x^{m+s} \binom{m}{s} f_{k-1}^{2s} f_{k-2}^{m-s} (1 - xf_k)^{-s-1}$$

$$= (1 - xf_k)^{-1} \sum_{m=0}^{\infty} x^m f_{k-2}^m \sum_{s=0}^{\infty} \binom{m}{s} \left\{ \frac{xf_{k-1}^2}{f_{k-2}(1 - xf_k)} \right\}^s$$

$$= (1 - xf_k)^{-1} \sum_{m=0}^{\infty} (xf_{k-2})^m \left\{ 1 + \frac{xf_{k-1}^2}{f_{k-2}(1 - xf_k)} \right\}^m$$

$$= (1 - xf_k)^{-1} \left\{ 1 - xf_{k-2} - \frac{x^2 f_{k-1}^2}{1 - xf_k} \right\}^{-1}$$

$$= \left\{ (1 - xf_k)(1 - xf_{k-2}) - x^2 f_{k-1}^2 \right\}^{-1}$$

$$= \left\{ 1 - x(f_k + f_{k-2}) + x^2(f_k f_{k-2} - f_{k-1}^2) \right\}^{-1}$$

$$= \left\{ 1 - x(F_{k+1} + F_{k-1}) + x^2(F_{k+1}F_{k-1} - F_k^2) \right\}^{-1} \quad (F_k \text{ is the } k^{th} \text{ Fibonacci number})$$

$$= \left\{ 1 - L_k x + (-1)^k x^2 \right\}^{-1} = \sum_{n=0}^{\infty} f_n^{(r)} x^n, \text{ by part (a).}$$

Comparing coefficients of the power series, this establishes part (b). N.B. $F_{(n+1)k} / F_k = f_n^{(r)}$.

Also solved by the Proposer.

Editorial Note: Dale Miller's name appeared incorrectly in H-237.

Continued from page 82. *******

Returning to (2) above, we can generate multigrades of higher orders. (For the standard method employed, see below.[1]) I give now, as an example, a third-order multigrade:

$$A_1F_1^m + A_2F_2^m \cdots A_{n-1}F_{n-1}^m + \left(\sum_1^{n-1} A - 2 \right) F_n^m + A_1(2F_n - F_1)^m + A_2(2F_n - F_2)^m \cdots A_{n-1}(2F_n - F_{n-1})^m$$

$$+ \left(\sum_1^{n-1} A - 2 \right) F_n^m = A_1(F_n - F_1)^m + A_2(F_n - F_2)^m \cdots A_{n-1}(F_n - F_{n-1})^m + \left(\sum_1^{n-1} A - 2 \right) 0^m$$

$$+ A_1(F_n + F_1)^m + A_2(F_n + F_2)^m \cdots A_{n-1}(F_n + F_{n-1}) + \left(\sum_1^{n-1} A - 2 \right) (2F_n)^m$$

(where $m = 1,2,3$).

[I have added F_n to each term in (2), and added the L.H.S. totals to the original R.H.S. and *vice versa.*] Expressed more tidily, the above becomes

$$A_1[(F_1)^m + (2F_n - F_1)^m] + A_2[(F_2)^m + (2F_n - F_2)^m] \cdots A_{n-1}[(F_{n-1})^m - (2F_n - F_{n-1})^m] + 2\left[\sum_1^{n-1} A - 2 \right] F_n^m$$

$$= A_1[(F_n - F_1)^m + (F_n + F_1)^m] + A_2[(F_n - F_2)^m + (F_n + F_2)^m] \cdots A_{n-1}[(F_n - F_{n-1})^m + (F_n + F_{n-1})^m]$$

$$+ \left(\sum_1^{n-1} A - 2 \right) [(2F_n)^m + 0^m] \qquad \text{(where } m = 1, 2, 3\text{)}.$$

Again, if we add any quantity B to each term, the final 0^m terms each become B^m.

REFERENCE

1. M. Kraitchik, *Mathematical Recreations,* George Allen & Unwin, London, 1960, page 79.