

*Proof.* By (4), (6) and (1) we have

$$\begin{aligned} \sum_{j=1}^k T_{k,j} L(j+n, p, q) &= \sum_{j=1}^k T_{k,j} \sum_{i=1}^q \prod_{l=1}^q \varrho_i^{\alpha_l} \sum_{h=0}^q c_h (p-h)^{j+n} \\ &= \sum_{i=1}^q \prod_{l=1}^q \varrho_i^{\alpha_l} \sum_{h=0}^q c_h (p-h)^n \sum_{j=1}^k T_{k,j} (p-h)^j = \sum_{i=1}^q \prod_{l=1}^q \varrho_i^{\alpha_l} \sum_{h=0}^q c_h (p-h)^n f(p-h). \end{aligned}$$

By definition  $p - h$  is an integer satisfying  $1 \leq p - h \leq p \leq k - 1$  and consequently by (1),  $f(p-h) = 0$  which proves the theorem.

#### REFERENCE

1. A. Ran, "One Parameter Groups of Formal Power Series," *Duke Math. J.*, Vol. 38 (1971), pp. 441-459.



[Continued from Page 48.]

Much more recently (1973), Jacobczyk [6] has given new iterative procedures for determining answers to both:

- (a) for each  $k$ ,  $1 \leq k \leq N$ , which will be the  $k^{\text{th}}$  place to be cast out?
- (b) for each  $k$ ,  $1 \leq k \leq N$ , when will the  $k^{\text{th}}$  place be cast out?

(The "Oberreihen" methods described by Ahrens also provide answers to both questions.)

#### REFERENCES

1. W. Ahrens, *Mathematische Unterhaltungen und Spiele*, 2nd ed., Leipzig, 1918, pp. 188-169.
2. C. G. Bachet, *Problèmes Plaisants et Delectables qui se font par les Nombres*, 1612.
3. W.W.R. Ball and H.S.M. Coxeter, *Mathematical Recreations and Essays*, London, 1939, pp. 32-35.
4. E. Busche, "Euber die Schubert'sche Lösung eines Bachet'schen Problems," *Math. Ann.*, 47 (1896), 105-112.
5. L. Euler, "Observationes circa novum et singulare progressionum genus," *Opera Omnia; Series Prima, Opera Mathematica*, Volumen Septimum, MCMXXIII, pp. 246-261.
6. F. Jakóbczyk, "On the generalized Josephus Problem," *Glasgow Math. J.*, 14 (1973), pp. 168-173.
7. Josephus, *The Jewish War*, III, 387-391. Translated by H. J. Thackeray, Loeb Classical Library, London, 1927. (See also Slavonic version cited in appendix.)
8. H. Schubert, *Zwölf Geduldspiele* (neue ausgabe), Leipzig 1899, pp. 120-132.
9. P. G. Tait, "On the Generalization of the Josephus Problem," *Proc. Roy. Soc. Edin.*, 22 (1898), pp. 432-435.

Sandy L. Zabell

